Measuring the rate of return on capital: What consequences for distribution, markup, task content and productivity in the US?

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Abstract

We propose a new measure of the rate of return on capital (RRK) for the United States from the early 1930s to 2019. We follow an augmented Euler equation, parametrize the model and use data on consumption, capital taxes, and asset management costs to produce the series. The estimation shows that the RRK trend is slightly negative while its level is comprised between 5% to 8% all over the period. Using these new data, we calculate rents, markups, and the change in labor and capital task content. We show that increasing rents and declining labor-based task share both contributed to the decline in the labor share over the past 20 years. Meanwhile, our estimation allows to produce a new measure of total factor productivity (TFP), the growth rate of which appears lower regarding previous estimations, especially after 2000. This confirms previous research showing that the concomitant rise in markup and tasks displacement does not concur with productivity gains and welfare improvement.

Keywords: Return on capital, Labor share, productivity, mark-up, automation
JEL Codes: E2, E4, J3, N12, O4

1 Introduction

The rate of return on capital (RRK henceforth) is a fundamental variable in most macroeconomic analyzes. However, its measure has been challenging despite its dramatic role and impact.

The traditional view on the RRK is due to Jorgenson (1963) and Hall and Jorgenson (1967), who link the locative cost of capital to the risk adjusted discount rate. As such, the RRK is the sum of the real (risk adjusted) interest rate, the depreciation rate and the expected inflation rate of capital assets (Caballero et al., 2017; Barkaï, 2021; Philippon and Gutierrez, 2022). However, using this relationship to measure the RRK poses three difficulties. First, it causes

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volatile and implausible estimation before the 1990s due to large variation in the real interest rates along with macroeconomic turmoil (Jordà et al., 2019; Karabarbounis and Neiman, 2019). This point is particularly questioning as it tends to disregard the consequences of monetary policy regarding the wedge between real interest rates and capital costs (Caballero et al., 2017; Fahri and Gourio, 2018). Indeed, Hall and Jorgenson formula is based on a long run persepective while bonds rate evolve in the short run following macroeconomic and political events.

Second, the use of interest rates ignores the consequences of financial regulation and capital controls, although both were large until the 1980s. Because financial markets were not as developed as they are today, the interest rates could not reflect the location cost of capital as supposed in Jorgenson (1963). This appears a major drawback for estimating the RRK in the long run, especially before the 1980s. Besides, the use of treasury yields to measure the risk free rate is questionable since bills are not totally riskless due to inflation exposure.

Third, it supposes data on the capital risk premium (KRP), the calculation of which dramatically depends on the estimation of the equity risk premium and a its large and diverging set of methodologies (Duarte and Rosa, 2015). In addition, equity valuation depends on the rent level since it affect firms' risk-adjusted net present value. In other words, this might be viewed as a rent entering the RRK estimation, which is what the calculation aims to avoid.

While Hall and Jorgenson (1967) model and the subsequent measure of the RRK focus on market return, our analysis proposes to focus on consumers' intertemporal choices, as proposed in Reis (2022a, 2022b) and Fahri and Gourio (2018). In this respect, we view the RRK as the opportunity cost of consumption given households intertemporal time preferences. Indeed, households are supposed to account for the RRK when they make their consumption choices in the long run, so, the related Euler equation can be used to calculate it. We thus parametrize the model and use data on consumption, taxes, and assets management costs to come up with the new measure.

The advantage of this calculation is threefold. First, it avoids the use of volatile interest rates and capital risk premium approximation. As long as one expects for the RRK to be rather smooth in the middle run, this measure offers better results. Second, theory shows that this measure does not include rents into the calculation of the RRK. So, we can disentangle capital costs from markups in the analysis. Third, data availability on consumption allows to build time series from 1929 onward, based on national account statistics. Fourth, since it does not impose any value to the technology parameters used in the production function, knowledge about the RRK can be used to measure those parameters. This point is particularly important as it allows to discuss and explain the recent change in some "big ratios" such as labor share and productivity decline.

The consequences of this calculation are twofold. First, this new database allows to deal with the issue of value added distribution. As soon as the RRK is known, a simple national account perspective can be used to distinguish capital share on one side and rents and markups on the other, as documented in Barkai (2021) and Gutierrez and Philippon (2022). In addition, relying on a CES production function, this allows to re-estimate the change in task content driven by labor and capital as done in Acemoglu and Restrepo (2019a) while taking markups into account. Therefore, we can quantitatively decompose the labor share change based on the evolution of markups, task content, and the neoclassical substitution effect. Second, using a simple Cobb-Douglas production function, we are now able to re-calculate the elasticity of the national production to capital and labor instead of fixing its value as in most analysis of growth accountability (Bergeaud et al., 2016). As a consequence, we can measure total factor productivity (TFP) based on the evolution of this elasticity. TFP evolution can hence be compared with the labor share decomposition to assess some of the recent transformation of the economy regarding growth and welfare.

Our main results are the following. First, our calculation does not produce large variations in the RRK in the medium run while its trend decline by 1pp in the 2000's. Second, we observe a decline in markups from 1950 to 1970 and a rise in rents after 2000. Meanwhile the share of capital as a pure cost is rather stable all over the period. Third, as long as we follow the literature and assume that the elasticity of substitution between labor and capital is inferior to one (Cf. Knoblach et al., 2020; Oberfield and Raval, 2021), our calculation documents a significant change in task content in favor of capital. Fourth, the decomposition of the labor share evolution from 1980 onward shows that most of the decline in the labor share since 2000 is due to the rise in markups, although the change in task content plays an important role too, especially in the business sector. Meanwhile, the neoclassical substitution effect tends to push the labor share up, therby preventing its value from declining too sharply after 2000. Those results are particularly consistent with Bergholt et al. (2022) results based SVAR methodology. Fifth, estimation of the decomposition of rents between workers and capital holders suggests that economic rents are increasingly distributed to capital holders. In other words, the decline in the labor share might be due to both the increasing monopolistic and monopsonistic positions of firms. Sixth, the extrapolation of total factor productivity shows that technical progress is lower than previously estimated and tends to decelerate. As a matter of fact, since TFP growth declines after 2000, this means that the concomitant rise in rents and tasks displacement has not been followed by a subsequent increase in innovation and growth (Aghion et al., 2019), while the related decline in the labor share tends to rise inequality and thereby decrease total welfare *ceteris paribus*.

The paper proceeds as follow. Section 1 presents the literature on factors share, rents and markups, change in task content and total factor productivity. Section 2 displays the model used to calculate the RRK and provides the related measure. Section 3 shows the consequences of this new measure on value added distribution between labor, capital and profit. Section 4 makes a decomposition of the change in the labor share based on the substitution effect, the change in markup, and the change in task content. Section 5 extends the analysis to total factor productivity. Section 6, proposes alternative measures. Section 7, adds discussion to the measure of the RRK. It accounts for alternative computation of the RRK, the distribution of rents between workers and capital holders, and the evolution for the whole economy. Section 6 concludes.

2 Production technologies and value added distribution

Along with the specific issue of the RRK, this study is linked to two strands of the literature. The first one concerns the evolution of the labor share in the long run. This is also linked to the literature on market power, rents and markups, the split of task content, and the neoclassical substitution between labor and capital. The second one relates to the calculation of total factor productivity. Along with a specific review of the literature, this section also provides the main concepts on which we will rely on all over the remaining of the paper, either for labor share change decomposition or TFP calculation.

2.1 Value added distribution

Although it proves to be stable during three decades, the labor share has declined in the US since the 2000s (Figure 1). Several theories thus emerged to explain what was considered until recently as a robust stylized fact (Bergholt et al., 2022; Grossman and Oberfield, 2022; Bazot and Guerreiro, 2023). Among them, three principal mechanisms have been discussed: the neoclassical substitution effect, the change in task content, and the rise in markups. In order to account for those elements, we first propose a simple model on which we will dwell on in this paper. Let us begin with a constant elasticity of substitution (CES) production function[1]:

$$Y = \left[(1-\alpha)^{\frac{1}{\sigma}}(A_L L)^{\frac{\sigma-1}{\sigma}} + \alpha^{\frac{1}{\sigma}}(A_K K)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \tag{1}$$

where α is a share parameter that may represent the share of tasks done with capital in the set of tasks required to produce the final good Y (cf. Acemoglu and Autor, 2011 or Acemoglu and Restrepo, 2018), $\alpha \in [0, \infty)$ is the capital-labor elasticity of substitution, and A_L and A_K are respectively the technical progress of labor and capital.

Beginning with the principle that firms have no influence on wages (W) and the RRK (R), and taking into account a potential markup ($\mu \geq 1$), cost minimization implies:

$$\frac{K}{L} = \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{A_K}{A_L}\right)^{\sigma-1}\left(\frac{W}{R}\right)^{\sigma} \tag{2}$$

From the first-order conditions, we can also deduce the labor share and capital share as a function of the ratios $\frac{L}{Y}$ and $\frac{K}{Y}$:

$$S_L = \frac{(1-\alpha)^{\frac{1}{\sigma}}}{\mu}\left(\frac{A_L L}{Y}\right)^{\frac{\sigma-1}{\sigma}} \tag{3}$$

$$S_K = \frac{\alpha^{\frac{1}{\sigma}}}{\mu}\left(\frac{A_K K}{Y}\right)^{\frac{\sigma-1}{\sigma}} \tag{4}$$

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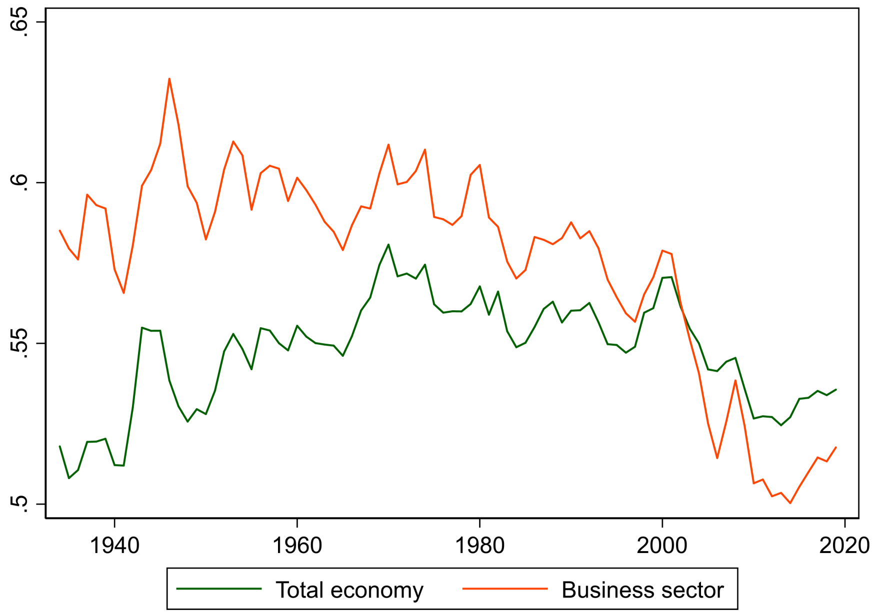
[1] It is worth noting that this model is a special case of the Bentolila and Saint-Paul model (2003). However, the aim of this model, based on the CES production function, is to easily account for the set of explanations used in the literature.



Note: the labor share is the ratio of wages bill to gross value added. For the business sector the denominator is net of production tax.

Figure 1: **Labor share in the US**

We can use (1) to obtain from (3) and (4) the labor share as a function of the quantities of factors:

$$S_L = \frac{1}{\mu} \frac{1}{1 + \left(\frac{\alpha}{1-\alpha}\right)^{\frac{1}{\sigma}} \left(\frac{A_K K}{A_L L}\right)^{\frac{\sigma-1}{\sigma}}} \tag{5}$$

$$S_K = \frac{1}{\mu} \frac{1}{1 + \left(\frac{1-\alpha}{\alpha}\right)^{\frac{1}{\sigma}} \left(\frac{A_L L}{A_K K}\right)^{\frac{\sigma-1}{\sigma}}} \tag{6}$$

The principal drawback here is that both equations fail to explain the labor share and capital share as a function of strictly exogenous variables. Going back to (2) we then obtain:

$$S_L = \frac{1}{\mu} \frac{1}{1 + \left(\frac{\alpha}{1-\alpha}\right) \left(\frac{\frac{W}{A_L}}{\frac{R}{A_K}}\right)^{\sigma-1}} \tag{7}$$

$$S_K = \frac{1}{\mu} \frac{1}{1 + \left(\frac{1-\alpha}{\alpha}\right) \left(\frac{\frac{R}{A_K}}{\frac{W}{A_L}}\right)^{\sigma-1}} \tag{8}$$

We can finally infer the profit share as:

$$\pi = 1 - S_L - S_K = 1 - \frac{1}{\mu} \tag{9}$$

Therefore, it can be seen from these equations that the labor share declines with markup (profit effect), the share of tasks done with capital (task content effect), and, if the elasticity of substitution is different from 1, with the ratio between labor cost and capital cost adjusted for factors' technical progress (substitution effect).

2.1.1 The substitution effect

The paper by Karababounis and Neiman (2014) provides a perfect illustration of the importance of the substitution. The authors claim that the fall in the relative price of investment goods is one of the main causes of the fall in the labor share in the United States and the majority of OECD countries. Intuitively, this explanation is similar to the effect of a change in factor costs illustrated in equation (7) as a rise in W/R reduced the labor share if $\sigma > 1$. Their analysis then shows that half of the observed fall in S_L results from the fall in the relative price of fixed capital. New technologies have therefore been paramount in reducing production costs and encouraging firms to invest.

Following the explanation based on the change in factor costs, analyzes documenting a decline in workers' market power makes use of similar mechanisms. The reduction of the union wage premium pushes wages to their competitive level (Bentolila and Saint Paul, 2003), so that the related decline in the cost of labor decreases the labor share if $\sigma < 1$. This coincides with the idea that the development of offshoring (Elsby et al., 2013; Dao et al., 2017) and the strengthening of international competition (Borjas et al., 1997; Autor et al., 2013) could be responsible for wages or employment stagnation over the past decades.

The increasing monopsonistic power of firms on the labor market has similar consequences on the labor share due to relative wage decline. Monopsonistic power might be due to increasing concentration of employment (Azar et al., 2020) or unions' declining power (Benmelech et al., 2020). In this respect, recent research finds that the U.S. manufacturing sector has been more monopsonistic since the 2000s (Yeh et al., 2022). On the other hand, using a model and estimating the related parameters based on within-firm-states, across market differences in wage and employment responses to state corporate tax changes in the U.S., Berger et al. (2019) show that labor market power has not contributed to the declining labor share.

By the same token, the reallocation of capital further to the deregulation of markets may have increased the rate of return on capital in relatively well endowed countries. The same goes for financial deregulation that reduces the cost of asset management (Bazot, 2022; Leblebicioglu and Weinberger, 2020). For instance, Caballero et al. (2017) and Fahri and Gourio (2018) use the computation of Gomme et al. (2011) (adjusted for the share of intangible capital (Koh et al., 2020)) to show that the rate of return on capital (based on net operating surplus) in the United States rose from 4% in 1980 to 6.5% in 2015.[2] This increase turns out to be higher than

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[2] Notice that the greater required return on capital may also be linked to expectation of increased risk or even to agents' increased risk-aversion.

the rise in average hourly wages in many sectors, thereby explaining the fall in the labor share if the sectors in question carry substantial economic weight and if $\sigma < 1$.
However, this explanation has to confront two pitfalls: (i) How to differentiate what is due to the remuneration of capital and what is due to economic rent? (ii) The rate of remuneration of capital (whether or not it includes capital gains) measured by Piketty and Zucman (2014) and Jordà et al. (2019) seems to have declined slightly since the 1970s–1980s in most OECD countries, suggesting a reverse mechanism. Fahri and Gourio (2018) and Eggerteson et al. (2021) propose to address those issues through the estimation of a macroeconomic model. In particular, Fahri and Gourio (2018) manage to measure the equity risk premium from the mid-1950's to 2010 through the calculation of target moments based on 11-years centered moving average rolling estimations. As such, their calculation shows that rising macroeconomic risks leads to a rising wedge between the risk free rate the RRK which pushes the labor share down.

2.1.2 The change in task content

Explanations about factor substitution are generally silent about the direct effects of technology on automation and production, and about the proportion of tasks associated with labor or capital. Models differentiating among tasks required for producing the final good (Zeira, 1998; Acemoglu and Zilibotti, 2001; Costinot and Vogel, 2010; Acemoglu and Autor, 2011) enable this limitation to be lifted. Y is the outcome of an assortment of tasks that are more or less open to substitution ranked by order of how readily they can be automated. In addition, the number of tasks may rise or fall with technology (Acemoglu and Restrepo, 2019a).[3]

The advantage of such modeling is to account here for the distribution effects between capital and labor in a way that is relatively independent of the elasticity of substitution. By taking up equations (3), (5), and (7) establishing the labor share in value-added, we can see that S_L is strictly negative in α. Automation thus tends to reduce the number of labor tasks (displacement effect) while productivity gains lead to the development of new tasks in which labor has a comparative advantage (reinstatement effect). If the displacement effect is greater than the reinstatement effect, then the number of tasks done with labor decreases, thereby reducing the labor share.

Task automation, however, is an endogenous process, and a wage increase (or capital cost decrease) may foster automation (Acemoglu and Restrepo, 2018; Martinez, 2019; Hubmer and Restrepo, 2022).[4] To properly understand the distinction between the classical capital-labor substitution effect and the automation effect, let's take the ageing population example. This leads to prioritizing leisure activities over consumption. So, the labor supply declines and wages rise. Two effects ensue. First, the quantity of capital used in production increases, the effect of which depends on the sign of $\sigma - 1$. Second, firms prime the automation of certain tasks via technology or innovation. The effect is unambiguous here as it does not depend on the

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[3] For example, the development of computers has enabled the creation of new jobs such as programmer or data scientist. Conversely, digital technology has meant the end for trades related to analogue devices.

[4] This is one of the possible explanations for the Industrial Revolution since a substantial wage increase is observed like never before or elsewhere in England from the mid-seventeenth eighteenth century (Allen, 2009). The rise in the relative costs of tasks associated with labor supposedly fostered mechanized production in the textile industry.

value of elasticity of substitution. This is why the labor share declines when this effect exceeds the neoclassical substitution effect. Therefore, aging of the population accelerates automation at the expense of wage-earners in areas that can be easily replaced by robots or machinery (Acemoglu and Restrepo, 2019b).

Moreover, as Dao et al. (2019) put forward, there is a connection between automation and offshoring of tasks. Some tasks can be more easily offshored than others, even if the costs of relocation prevent generalization of the phenomenon. Thus, a relative increase in wages leads to the outsourcing of certain tasks. Likewise, firms have an interest in offshoring the most labor intensive tasks so as to reduce their costs (cf. Feenstra and Hanson, 1996; Elsby et al., 2013). This is why the offshoring of tasks cannot have beneficial effects on the demand for labor unless the fall in the underlying production cost is great enough to increase output of the final good.

2.1.3 Rents and markups

The standard strategy to measure markups at a national level is to take the profit as a residual after the payment of other costs. A first response that the literature provides is to approximate R. In this respect, Barkai (2020)[5] suggests putting a figure on the share of capital by relying on the theoretical model developed by Jorgenson (1963) and applied by Hall and Jorgenson (1967). Under these assumptions, the results show a fall in both the labor share and the capital share in the United States between 1984 and 2014. This double reduction can only be explained by a rise in economic rents.[6]

Gutiérrez and Philippon (2022) extends Barkai's work to a set of European economies from 1995 to 2015. The two approaches are very similar in terms of method, especially when it concerns the computation of the capital share. However, Gutiérrez and Philippon (2022) calculation is more precise as they do not use a fixed equity premium, and use industry and firms level data (Compustat) for their computation. Whereas the labor share declines to the benefit of economic rents in the United States, Gutierrez and Philippon (2022) shows that this finding cannot be transposed to European countries.

However, analyzing rents on aggregate data is not without its major problems. Karabarbounis and Neiman (2019) show that the non-allocated share of value-added, which they label "factorless income", is too easily associated with rents. As a matter of fact, their principal critics to this approach stands to the large volatility of the estimation of R.

A second, more microeconomic approach draws on disaggregated data to estimate production functions in order to deduce individual (or sectoral) markups and the aggregate markup. Here, there is no need to assign compensations to the different production factors, dispensing with determining R. As such, Hall (2018) econometrically estimates the production function by sector by generalizing Solow's decomposition to imperfect competition. The results show that, the markup rose from 1.12 to 1.38 in the US from 1988 to 2015. In another register, De Loekker,

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[5] It is the working paper dated 2016 that really launched the macroeconomic approach in the literature.

[6] For the sake of thoroughness, it should be noted that Rognlie (2015) was the first to dissociate the share of economic rents and of capital. However, he used the reverse approach to Barkai: after setting the level of rents, he inferred plausible values of R and/or μ.

Eeckhout, and Unger (2020) propose to estimate an elasticity of production relative to one of the production factors, and then, using a few straightforward computations, to quantify the markup. The results exhibit a surge of the markup rate in the United States between 1955 and 2016, rising from 1.21 to 1.6. Basu (2019) emphasizes that the margin levels reported by these two studies are probably overestimated. A possible explanation put forward by Foster et al. (2022) and Hubmer and Restrepo (2022) for such results is that technological change is imperfectly taken into account. As such, relying on a more granular dataset based on establishment level observations Foster et al. (2022) document a larger flexibility of output elasticities (consistent with changing production technologies), and lower markups. As such, their result suggests that markups may have not increased significantly for US manufacturing since the 1970s.

2.2 Technology and total factor productivity

The measure of total factor productivity at the aggregate national level is based on an intensive Cobb-Douglas production function ($y = Ak^{\alpha}$). As such, information about labor productivity and capital intensity is enough to calculate A as long as α is known. For that reason, α is either set to 1/3, or to 0.3 (as in Bergeaud et al., 2016) or to the share of capital in value added – which tacitly needs to assume perfect competition.

In this respect, additional information on the RRK allows to produce a direct measure of α based on the basic $\frac{MPK}{R} = \frac{MPL}{W}$ formula without additional assumption. In other words, becausethe system is under-determined, the only way to propose a measure of both α and μ is to know about the value of R. As we will see, proceeding this way may have dramatic consequences onthe measure of TFP. Regarding this, our paper agrees with recent researches on the additive nature of TFP growth (Philippon, 2023).

3 A new measure of the rate of return rate on capital

3.1 The basic framework

As shown in the literature review, the RRK is a key variable to understand value added distribution and to measure rents based on a macroeconomic approach. Here, we propose to rely on the intertemporal consumption choice, as in Reis (2022a, 2022b) and Fahri and Gourio (2018), to measure the RRK. The advantage of this method is four folds. First, it allows to produce series until the 1930s, while Gutierrez and Philippon's (2022) measure starts in 1992 and Barkai's (2020) measure start in 1985. Second, it does not depend on the choice of the reference rate. Third, it has not to rely on a previous measure of the capital risk premium. Fourth, unlike interest rates and equity premium, its measure is barely subject to macroeconomic turmoil. However, because it is an indirect measure which relies on the choice of parameters value, this measure is complementary to the ones produced hitherto. As such, comparison with Barkai (2020) and Gutierrez and Philippon (2022) for the recent period, when turmoil is low, matter.

3.2 The intertemporal consumer choice

The aim of this model is to produce exploitable results from a pure empirical perspective. In this respect, we keep it as simple and general as possible. We discuss the related hypotheses, in particular the choice of a non-cumulative utility function, in section 7. So, let's consider a representative consumer, which the intertemporal utility function is defined by:

$$U = \int_0^{+\infty} u[c(t), l(t)] \exp\{-(\rho - n)t\} \, \mathrm{d}t \tag{10}$$

With $c(t)$ the level of consumption at time t, $l(t)$ the household leisure at t, ρ the rate of time preference and n the growth rate of population. Note that the utility of leisure and consumption can be additive or not.

The representative household intertemporal budget constraint accounts for taxation and financial intermediation costs, so that:

$$\begin{aligned} \dot{a}(t) = (1 - \upsilon_w(t))W(t) + (1 - \upsilon_\pi(t))\pi(t) + \\ [(1 - \upsilon_r(t))R(t) - \varphi_h(t)]\, a(t) - (1 - \upsilon_c(t))c(t) + \nu - na(t) \end{aligned} \tag{11}$$

With R the rate of return on assets a, φ_h the cost of asset management for households and υ_w, υ_π, υ_a and υ_c the tax rate on wages, profits, assets return, and consumption, and ν the government transfer to households. $na(t)$ takes into account the impact of the growth rate of population on the stock of assets.

$Ra(t)$ is the amount transferred to investors for holding assets. Since the representative household holds all the assets, R is the return to a \$1 worth portfolio composed of a weighted set of assets available in the economy. It is assumed that the representative agent is price taker, so, she has no impact on the rate of return.

It is also important to distinguish between income from assets and profit. The former is paid proportionally to the amount of assets invested by the household. The latter is what remain once labor and capital costs are paid. In other words, although firms' profit can be delivered to shareholders in function of their detained shares, the total amount of distributed profits does not depend on the volume of assets. This explains the distinction we make in equation (2). For that reason, the RRK calculated here does not include rents as long as markups are independent from total assets (which is true at the macroeconomic level).

The intertemporal maximization provides the related Euler equation:

$$R(t) = \frac{\rho + \varphi_h(t) + \dfrac{\dot{\upsilon}_c(t)}{1 + \upsilon_c(t)} - \left[\dfrac{u_{cc}[c(t), l(t)]c(t)}{u_c[c(t), l(t)]}\right] \dfrac{\dot{c}(t)}{c(t)}}{1 - \upsilon_a(t)}$$

Since $\gamma \equiv -\frac{u_{cc}[c(t),l(t)]c(t)}{u_c[c(t),l(t)]}$ correspond to the relative risk aversion and $\frac{\dot{c}(t)}{c(t)}$ is the growth rate of consumption (g_c), this becomes:

$$R(t) = \frac{\rho + \varphi_h(t) + \dfrac{\dot{\upsilon}_c(t)}{1 + \upsilon_c(t)} + \gamma g_c}{1 - \upsilon_a(t)}$$

This can be seen as the result of an augmented Ramsey law. Note that the same implementation in discrete time states:

$$R_t = \frac{(1+\rho)(1+\varphi_{h,t}) + \left(1 + \frac{\Delta \upsilon_{c,t}}{1+\upsilon_{c,t}}\right) + (1+g_c)^{\gamma}}{1-\upsilon_{a,t}} - 1$$

3.3 Multiple households

As in Reis (2022b) we would certainly like to account for consumers' heterogeneity, especially because a fraction θ of hand-to-mouth households consume their entire labor income. Therefore, since these households are not sensitive to the RRK, this leads to a biased estimation of R. Because consumption is composed of two households' types, the growth rate of consumption is now given by the following formula:

$$g_c - \theta g_{y,l} = (1-\theta)\left((1-\upsilon_a(t))R(t) - \rho - \varphi_h(t) - \frac{\dot{\upsilon}_c(t)}{1+\upsilon_c(t)}\right)\gamma^{-1}$$

With $g_{y,l}$ the average growth rate of labor income. Thus, the RRK can be easily derived as:

$$R(t) = \frac{\rho + \varphi_h(t) + \frac{\dot{\upsilon}_c(t)}{1+\upsilon_c(t)} + \frac{\gamma(g_c - \theta g_{y,l})}{1-\theta}}{1-\upsilon_a(t)} \tag{12}$$

Equation (12) is essential for our measurement exercise as it provides the basic formula on which we rely on to assess the RRK.

3.4 The calculation of the RRK

In order to measure the RRK we need fixed values for two parameters, namely, the relative risk aversion and the rate of time preference. The rate of time preference is assumed to be equal to 0.02 as in most calibration exercise. The rate of risk aversion is set based on meta-analyses about the elasticity of intertemporal substitution (EIS) and the relative risk aversion (RRA). As such $\gamma = 2$ coincides with most estimations of the EIS (Havranek et al., 2015) while $\gamma = 1$ coincides with estimation of the RRA (Elminejad et al., 2023).[7] However, note that since this parameter is assumed to be fixed, this mainly plays on the level of the RRK and barely affect its trend. As such, insofar as we are interested in the change of the variables of interest, this choice is not of large importance.

The growth rate of consumption is obtained from the Bureau of Labor Statistics, the capital income taxation is from Piketty and Zucman (2014) and McDaniel (2007), the cost of asset management is based on Philippon (2015).[8] We use Kaplan et al. (2014) results on the share

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[7] It is worth noting that both measures of IES and RRA are volatile across studies and strongly depends on the choice made to measure the real rate of return to capital (Hall, 1988).

[8] Here we assume that the management cost of real estate is equal to the management cost of financial assets. In this respect, it is worth noting that housing only represents 20% of all investable assets. In addition, because financial cost can be supported on both asset (mostly households) and liability side (mostly enterprises), we assumed that asset management costs devoted to households represents half of total costs. The other half is then supported by enterprises.

of hand-to-mouth households in the US to set $\theta = 0.3$. Lastly, Because consumption may vary stochastically or due to sudden change in consumer expectation, we use smoothing techniques to remove this "noise" from the calculations.[9] This point is discussed in section 7 as we look at the consequences of expectations on the calculation.

Figure 2 shows that the RRK decline from 1950 onward, whatever the parameters used for calculation. In the details, depending on γ, we see that the RRK stays around 9% or 13% from the 1950's to the early 1970's then declines during the 1970's to stay around 8% or 12% from 1980 to 2000. Then, we observe a drop in the mid-2000s. It is worth noting that the series is not volatile in the medium run, which contrasts with previous estimations based on Hall and Jorgenson (1967) model. In addition, our estimation produces on average higher cost of capital than Barkai (2019). Once we add the depreciation rate, we observe a very stable cost of capital, around 10% to 15% depending on the assumption made about relative risk aversion.

Note: The rate of return on capital is based on Euler equation (12) with two different measures of relative risk aversion (γ)

Figure 2: **The rate of return on capital**

4 Consequences on markup, task content and distribution

As long as the RRK is known, it is quite easy to decompose value added into labor, capital and profit share. This eventually allows to measure rents and markups. In addition, it is possible to decompose the evolution of the labor share calculated in equation (7) to measure the change in task content, as done in Acemoglu and Restrepo (2019a). The main difference here is that

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[9] We use lowess smoothing method with a bandwidth of 0.2 to allow short term variation.

we do not have to assume perfect competition to produce this measure, what can substantially affect the results. Lastly, we can use the related decomposition to explain the recent decline in the labor share according to the change in markup and task content and the neoclassical substitution effect.

4.1 Markups and capital share

Labor and capital share can be calculated from the following set of equations:

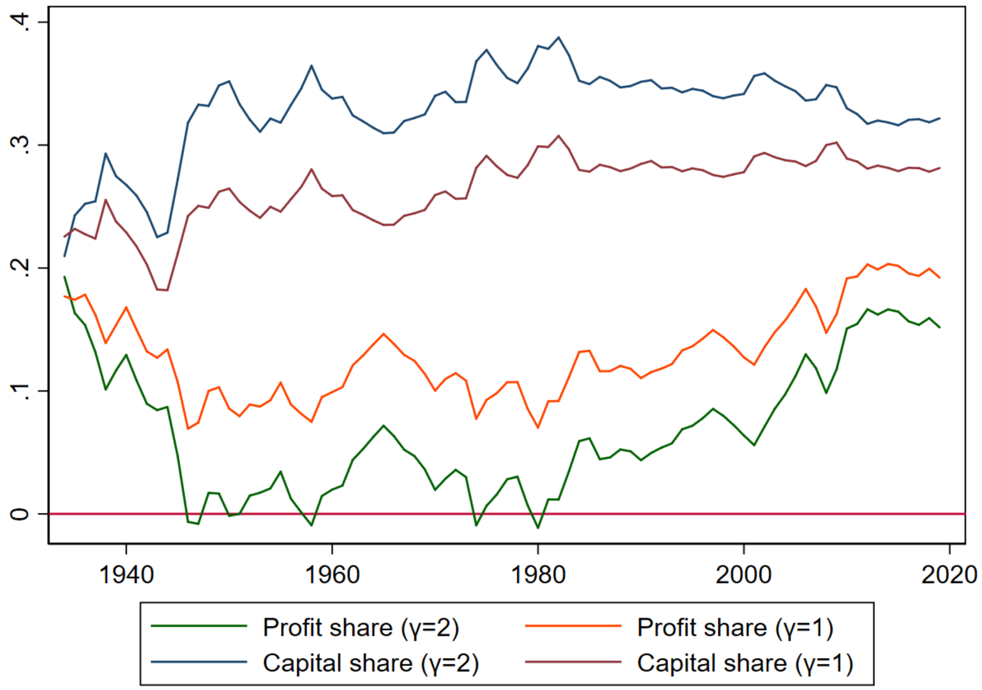
$$S_L = \frac{WL}{PY} \tag{13}$$

$$S_K = \frac{(R + \delta + \varphi_h)P_K K}{PY} \tag{14}$$

Equation (13) simply states that the labor share is the ratio of wage bill to value added. However, one might suspect wages to depend on rents in the sense that profits can be shared with employees; so, a part of wages would be due to firms' market power. The labor share is not adjusted for this effect here, but we discuss this point below in section 6. Equation (14) shows that the capital share is the ratio of total return on capital to value added. $R+\delta+\varphi_h$ represents the cost of using capital for production and accounts for the rate of return, the depreciation, and the financial intermediation cost. In other words, the cost for the user of fixed capital is equal to what she would gain if she invested the related capital on the market plus fixed capital costs. This coincides with the rate of return obtained in equation (12) to which we add $\delta + \varphi_h$. We can finally compute firms' profit rate and markup from equation (9). We focus here on the business sector as this sector for two reasons. First, accounting issues are less important due to mixed income and real estates income. Second, this sector is more subject to markup.

The calculation on the distribution of value added between profit and capital shares leads to Figure 3. We see that both profit and capital shares display low volatility from 1960 onwards. In the details, the capital share is on average close to 30% of value added over the past 50 years. Its trend is positive until 1970 then its level declines slightly until 2019 when $\gamma = 2$ but remains very stable when $\gamma = 1$. The share of profit decline significantly from 1935 to 1950, it remains stable until 1980, and finally increases (especially after 2000), to reach about 15% of value added. This suggests that while the rise in markups have pushed the labor share downward over the past three decades, the capital share barely affect the labor share since 1970. In other words, capital and labor share of income do not display opposite pattern, as assumed in Piketty (2013). It is lastly worth noting that the pattern of the most recent years suggests that rents tends to stabilize since 2015, thereby explaining the labor share going slightly up over the past 5 years (Cf. Figure 1).

This result confirms the negative role played by rents on the labor share, it thus coincides with recent evaluation of profits from Barkai (2020). In fact, the RRK sharply declines from 1985 to 2015 along with the interest rate trend drop in Barkai's estimation. However, Barkai's measure dwells on corporate bonds yield and the Weighted Average Cost of Capital (WACC), which value evolves dramatically with the volatility of interest rates, as claimed in Karabarbounis



Note: The capital share is calculated from the ratio of capital spending to GVA (see eq. (14)). The profit share is what remain once the labor share and the capital share are removed (eq. (9)). The calculation includes two different measures of relative risk aversion (γ) following meta-analyses on EIS and RRA.

Figure 3: **Profit and capital share of the value added**

and Neiman (2019). Here, our measure confirms that this volatility is not the cause of the related rise in profit rate.

Comparison with Gutierrez and Philippon (2022), who follows Barkai's (2020) methodology, is even less contrasted. The reason is that Gutierrez and Philippon (2022) focus on a shorter period (1989-2015), thereby avoiding the turmoil of the 1980's. In addition, their calculation use industry instead of aggregated data. As a result, although the markup is slightly higher in Gutierrez and Philippon (2022), our series appear very similar, displaying the same positive trend. In other words, our estimation of the markup trend is close to the one produced in Gutierrez and Philippon (2022) based on compustat data for the 1989-2015 period.

4.2 Change in task content

Because of the CES complexity, we cannot produce a direct measure of α. However, we can measure the change in task content in favor of capital based on the derivation of equation (7). This leads to the same equation displayed in Acemoglu and Restrepo (2019a) except that we can now account for the effect of markup in the calculation. After defining the reference year

t_0, the change in the share of task produced with labor is obtained from:

$$\text{Change in task content} = \underbrace{\ln S_{L,t} - \ln S_{L,t_0}}_{\text{change in labor share}} \underbrace{- \ln\left(\frac{1}{\mu_t}\right) + \ln\left(\frac{1}{\mu_{t_0}}\right)}_{\text{change in markup}} - \underbrace{(1-\sigma)(1-S_{L,t_0})\left[\ln\left(\frac{\frac{W_t}{R_t}}{\frac{W_{t_0}}{R_{t_0}}}\right) - g_A\right]}_{\text{substitution effect}} \tag{15}$$

With g_A the growth rate of A_L/A_K. Here we follow Acemoglu and Restrepo (2019) and assume that this ratio grows at the same rate as labor productivity. Empirically, most estimates of σ turn out to be less than 1 (Acemoglu, 2003; Oberfield and Raval, 2021; Raval, 2019). A recent analysis using meta-regressions on U.S. data (Knoblach et al., 2020) indicates that $\sigma \in [0,45; 0,87]$ even if some heterogeneity is observed depending on the estimation methods. As such we chose $\sigma = 0.8$, as assumed in most calibration exercise.

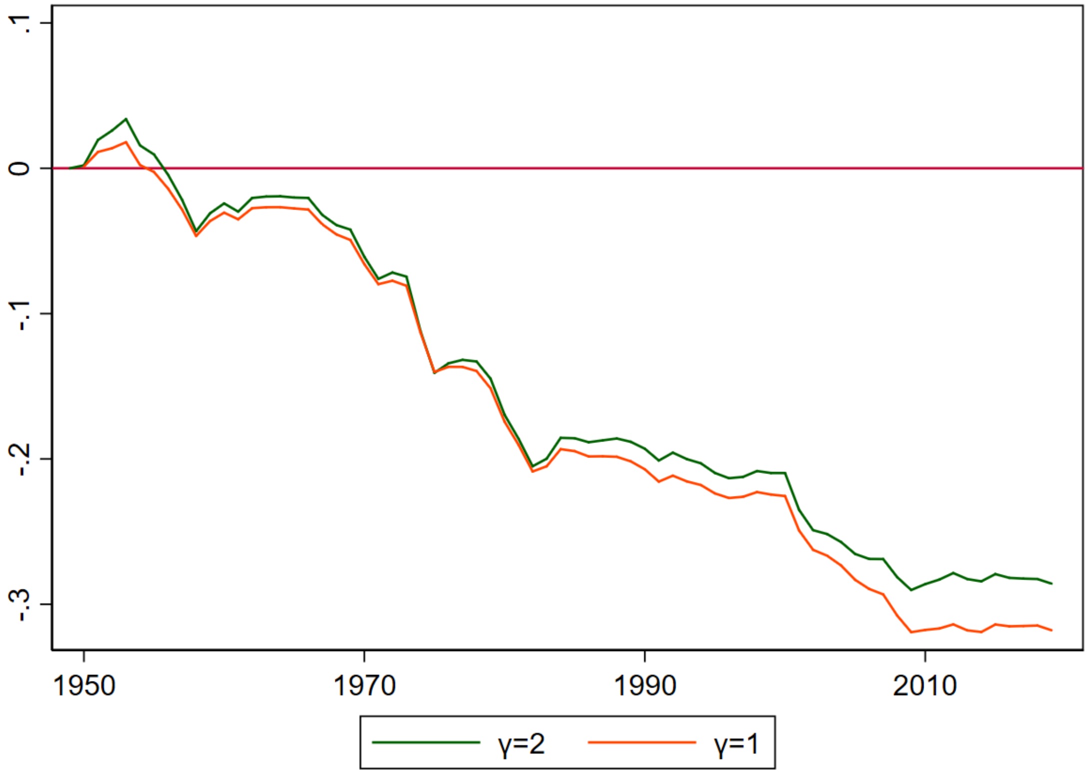
The use of equation (15) leads to Figure 4. We see that from 1948 to 2019, the share of tasks done with labor declines substantially around 30pp. This means that a large number of tasks has been automated or offshored and that the creation of new tasks did not offset this effect. In the details, the share of labor based tasks declines sharply from the 1970 to 1980, declines slightly from 1980 to 2000, declines strongly again from 2000 to 2010, and stagnates from 2010 onward. It is worth noting that the decline observed after 2000 coincides with China adhering to the WTO but also with the development of numerical methods and artificial intelligence.

Unlike Acemoglu and Restrepo (2019a) our calculations display a positive change in task content in favor of capital-based tasks from 1960 to 1980. The main reason for this result is that our calculation accounts for markups. Ignoring markups prevents from capturing the positive effect that rising competition had on the labor share from 1960 onward, which ultimately leads to the underestimation of the change in task content. However, while the change in task content is associated with an important increase in labor productivity before 1980, it is associated with less vivid productivity gains from 2000 onward. This thereby suggests that the change in task content is not of the same nature during both periods. As shown in Acemoglu and Restrepo (2018), so-so automation after the 2000s might be responsible for this dramatic difference.

4.3 The effect of technology in CES production function

The CES production function is probably the best compromise between reality and tractability. However, as shown previously, the calculation in the change in task content must rely on some hypothesis about the value of g_A which may slightly affect the results. Instead, we can propose an alternative measure of the effect of technology "as a whole". Following this framework, the economic production is represented by:

$$Y = \left[\Gamma L^{\frac{\sigma-1}{\sigma}} + \Omega K^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$



Note: The change in labor share task is calculated from eq. (15). The calculation includes two different measures of relative risk aversion (γ) following Chetty (2006). The elasticity of substitution between labor and capital is fixed to 0.8. g_A (the growth rate of A_L/A_K) is assumed to be equal to the growth rate of labor productivity.

Figure 4: **Change in the share of labor based tasks**

With $\Gamma = (1-\alpha)A_L^{\frac{\sigma-1}{\sigma}}$ the labor based technology and $\Omega = \alpha A_K^{\frac{\sigma-1}{\sigma}}$ the capital based technology. Here, we cannot distinguish between the effect of labor and capital technological bias (A_L/A_K) on one side and the effect automated tasks (α) on the other. We capture both technological effects at the same time provided that firms have no influence on wages (W) and the RRK (R). From the first-order conditions, we then have:

$$\frac{\Omega}{\Gamma} = \frac{S_K}{S_L}\left(\frac{K}{L}\right)^{\frac{1-\sigma}{\sigma}} \tag{16}$$

We can thus use the ratio Ω/Γ to know about the relative evolution of technology—whether it is labor or capital based. If this ratio increases, this means that the technology tends to increase the weight of capital relative to labor in national costs. Following this we can calculate the share of capital based technology ω from[10]:

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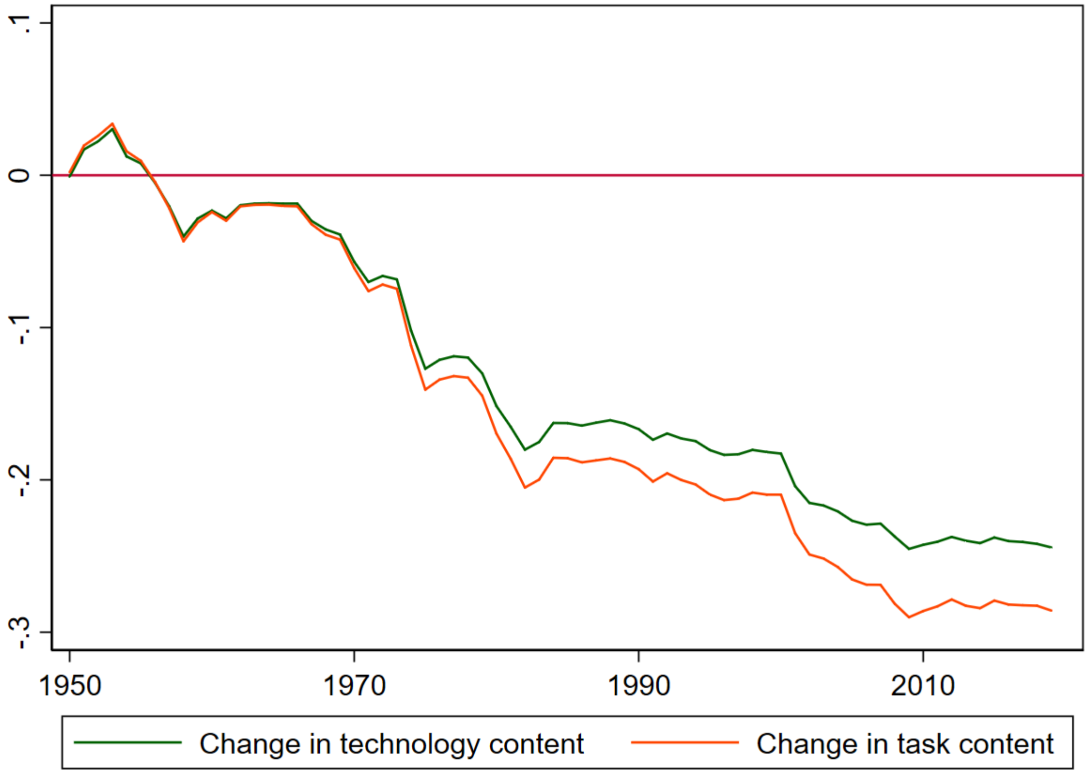
[10] Another possibility would be to assume Hicks neutral technology. In such a case $\frac{A_L}{A_K} = 1$ and the share of tasks follows from: $\alpha = \frac{\Omega/\Gamma}{1+\Omega/\Gamma}$

$$\omega = \frac{\Omega/\Gamma}{1+\Omega/\Gamma} \tag{17}$$

Therefore, we can use (17) to measure the change in technology content and compare it with the change in task content obtained from (15).

Figure 5 shows that the evolution of both series follows a very similar pattern, suggesting that most of the change in technology content is due to the change in task content. In other words, the hypothesis made about g_A did not dramatically affect the change in task content measured previously. However, and quite interestingly, ω decreases less than α. This suggests that part of the change in task content is soaked up by the change in $\frac{A_L}{A_K}$.

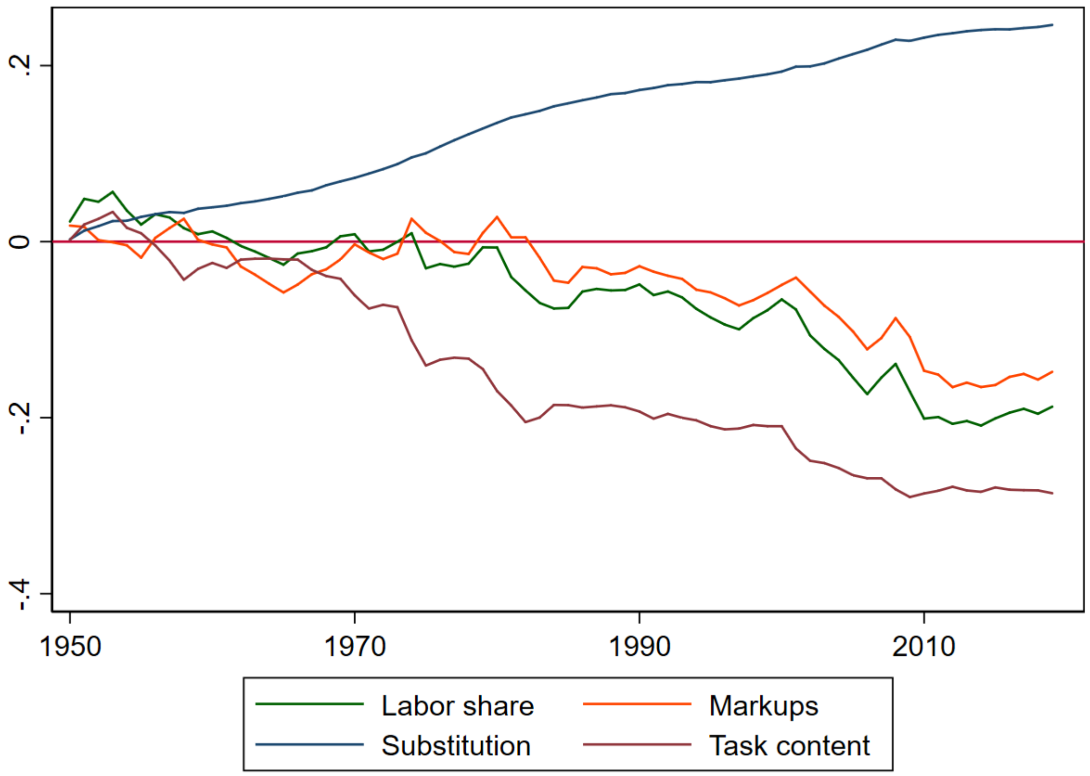


Note: The change in task and technology accounts for both task change and labor and capital technology change at the same time based on eq. (16). The relative risk aversion is fixed to 2 while the elasticity of substitution between labor and capital is fixed to 0.8.

Figure 5: **Change in task and technology content**

4.4 Decomposing the labor share decline

We can finally use equation (15) to decompose the decline in the labor share. Three main facts come out (Figure 6). First, the substitution effect tends to increase the labor share. This directly falls from the hypothesis of complementarity between factors ($\sigma < 1$). Since, the capital output ratio increases over this period, the concomitant rise in labor to capital costs more than offsets the effect of capital accumulation on the share of capital. In other words, the labor share would have decreased even more should the price effect had not dominated the quantity effect and pushed the capital share down. Second, the change in markup is not as large as the labor share decline. Therefore, the development of rents may have had important consequences, especially after 2000, but it cannot explain the whole decline in the labor share. Third, the change in task contents is larger than the change in the labor share. If we compare with markups, the decline in the labor share over the past 40 years is due to the change in task content for about half of the total. As such, it is worth noting that markup plays a stronger role than task content if we focus on the post-2000 period. However, one might argue that the change in task content as well as the substitution effect are two sides of the same coin as they are directly linked to the relative rise in wages compared to capital costs (Hubmer and Restrepo, 2022). If so, the net effect of task content should account for the positive substitution effect. Following this hypothesis, three quarter of the labor share drop since 1950 is explained by the change in markups. This figure is close to one if we focus on the post-1980 period.



Note: The labor share change is decomposed based on eq. (15) to account for the change in task content, markup and capital-labor substitution effect. g_A (the growth rate of A_L/A_K) is assumed to be equal to the growth rate of labor productivity. The relative risk aversion is fixed to 2 while the elasticity of substitution between labor and capital is fixed to 0.8.

Figure 6: **Labor share change decomposition**

5 An extension to total factor productivity

A fundamental question behind the concomitant evolution of the labor share, task automation and rising markups is their link with productivity and growth. So far, we used a CES production function, however, it proves intractable as one wants to measure TFP from Solow decomposition. Therefore, following the literature, we use a Cobb-Douglas production function. Although this may be seen as a necessary simplification, recent analysis has shown that the economy might be close to this function in a very long run perspective (León-Ledesma and Satchi, 2019).

If the production function is a Cobb-Douglas, the share of labor is equal to the elasticity of the output to labor when competition is perfect. Nonetheless, as long as one introduces a markup, the elasticity diverges. In most measures of total factor productivity, this elasticity is either set equal to the labor share or given arbitrary. Because we are now able to measure markups, we can easily infer the elasticity of production to inputs, and thereby use this value to calculate TFP. So, if:

$$Y = F(K, L) = AK^{\alpha}L^{1-\alpha} \tag{18}$$

Given that prices equal marginal productivity we come up with the elasticity of production to capital and labor based on the first order condition:

$$\alpha^{CD} = \mu(R + \delta)\frac{K}{Y} \tag{19}$$

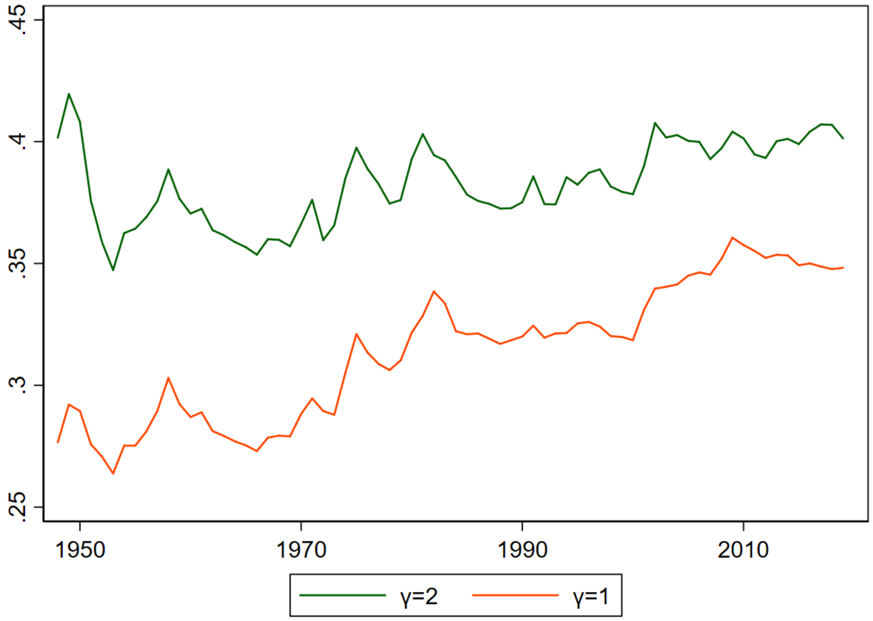
We thus see from Figure 7 that α^{CD} increases dramatically since 1960. This concurs with the increasing share of task associated to capital documented above.

Because it is not fixed, the use of α^{CD} can steadily affect the calculation of TFP. Let's inquire this in details. From (18) we have $y = Ak^{\alpha}$ with $y \equiv Y/L$ and $k \equiv K/L$, so TFP is obtained from:

$$A^{CD} = \frac{y}{k^{\alpha^{CD}}} \tag{20}$$

Then, we use the BEA data to produce the related series. Figure 8 displays the series and compares it to the value produced based on the same assumption used in Bergeaud et al. (2016). Because α^{CD} is not fixed and slightly diverges from 0.3 over the period, a significant and increasing gap appears between both series up to the early 1970's. In other words, productivity growth over the past 50 years appears lower than assumed so far.

In order to get further insights on the long term trend of TFP growth, we used lowess smoothing techniques to compare the growth rate of each TFP series. It comes from Figure 9 that all series decline continuously, suggesting that TFP growth in a near future will not get back to the value of the golden age. By comparison with $\alpha = 0.3$, our calculation displays lower growth rate since 1960. This suggests an overestimation of TFP growth during the golden age. As such, capital accumulation may have played a bigger role during this period than previously



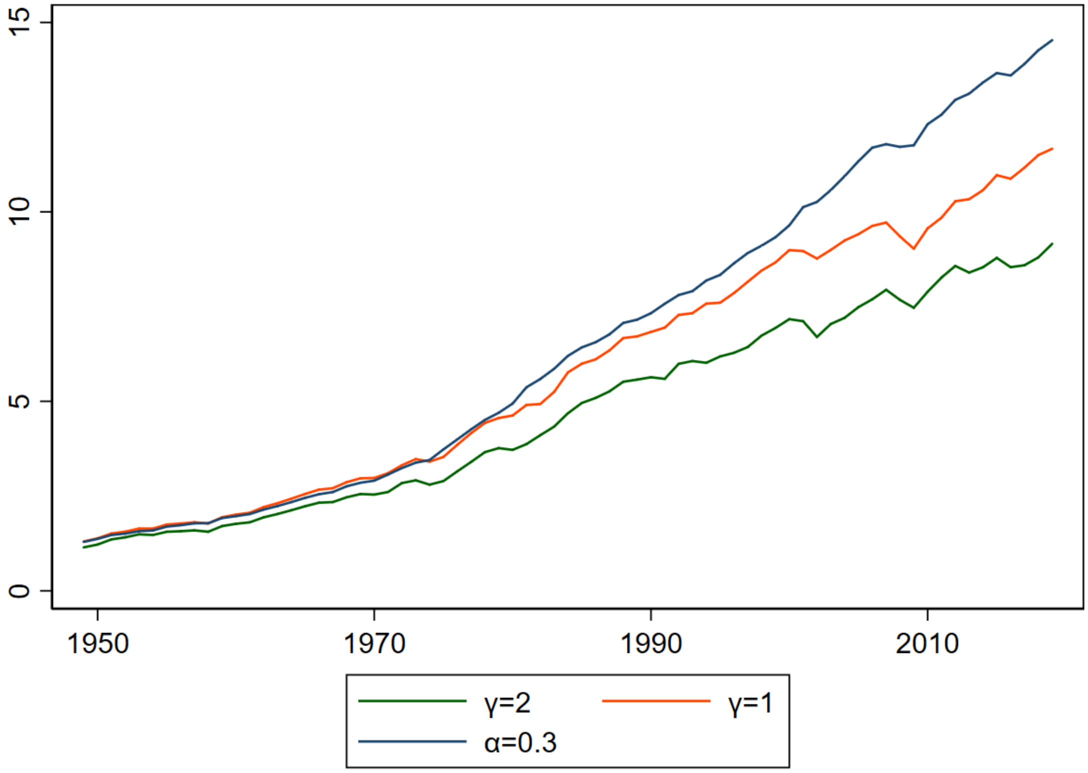
Note: The elasticity of output to capital is calculated from eq. (19) and is based on a Cobb-Douglas production function. The calculation includes two different measures of relative risk aversion (γ).

Figure 7: **Calculation of α with a Cobb-Douglas production function**

assumed based on classical Solow-growth decomposition. In addition, the computer age displays relatively small TFP growth. The most recent period shows very low productivity gains, which concurs with low GDP growth observed over the past 20 years. In this respect, it is worth noting that this gap is linked with the rise in markups and the change in tasks content documented previously. Therefore, if one assumes that the markup comes from market power due to innovation or firms size (Autor et al., 2020; Kehrig and Vincent, 2021), this concurs with lower TFP growth. The same holds true regarding the increasing use of digitalization, although this may be due to the fact that so-so automation does not generate productivity gains (Acemoglu and Restrepo, 2018). Another explanation might be that market power plays a decisive role in reducing TFP growth due to changes in allocation efficiency (Baquee and Fahri, 2020).

6 Alternative measures

In this section we discuss two potential alternatives to the calculations produced so far. First, we look at the consequences of the use of an extended calculation making distinction between safe and risky assets. Second, we discuss the consequences of rents distribution between workers and capital holders.



Note: The figure compares TFP with $\alpha = 0.3$ from Bergeaud et al. (2018) to TFP based eq. (20). The calculation includes two different measures of relative risk aversion (γ).

Figure 8: **Total factor productivity measure**

6.1 The RRK computation based on the classical Ramsey law

Instead of relying on (12) to measure the RRK, we follow Reis (2022b) and distinguish between two types of households among non hand-to-mouth households. A fraction these households invests in a risky project, the rate of return of which is $m > R$, while the other fraction invests in government bonds for a return $r < R$. In this respect, the rate of return obtained from (12) corresponds to the weighted average return on both type of investments, so that: $R = \frac{am+dr}{a+d}$, with a the amount of risky assets invested and d the amount of safe assets.[11] So, the RRK on risky assets can be computed from:

$$m = r + \left(1 + \frac{d}{a}\right)(R - r) \tag{21}$$

Using government bills return from Jordà et al. (2018) data we come up with m in figure 10. We see that both the markup trend and pattern are similar, though the rise is a bit smaller with the risky estimation. For that reason relying on the estimation of m produce similar decomposition of the labor share change and similar TFP growth. In addition, it is worth noting

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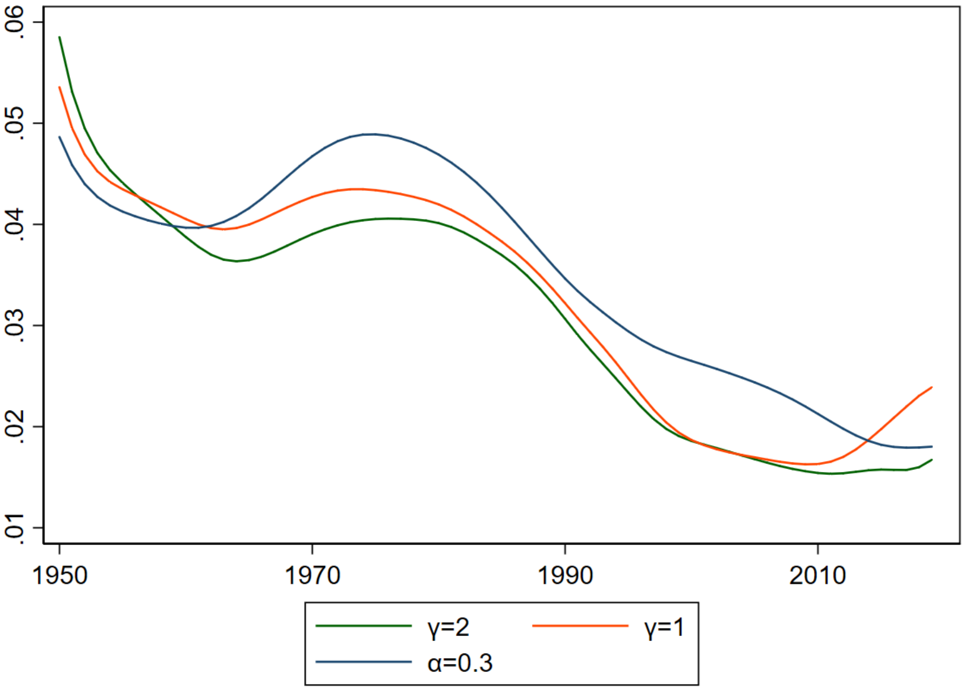
[11] The interested reader can refer to Reis (2022b) for a theoretical specification of this result.



Note: The figure compares TFP growth with $\alpha = 0.3$ from Bergeaud et al. (2018) to TFP growth based eq. (20). A lowess smoothing technique of band width equals to 0.5 has been applied to the series. The calculation includes two different measures of relative risk aversion (γ) following Chetty (2006).

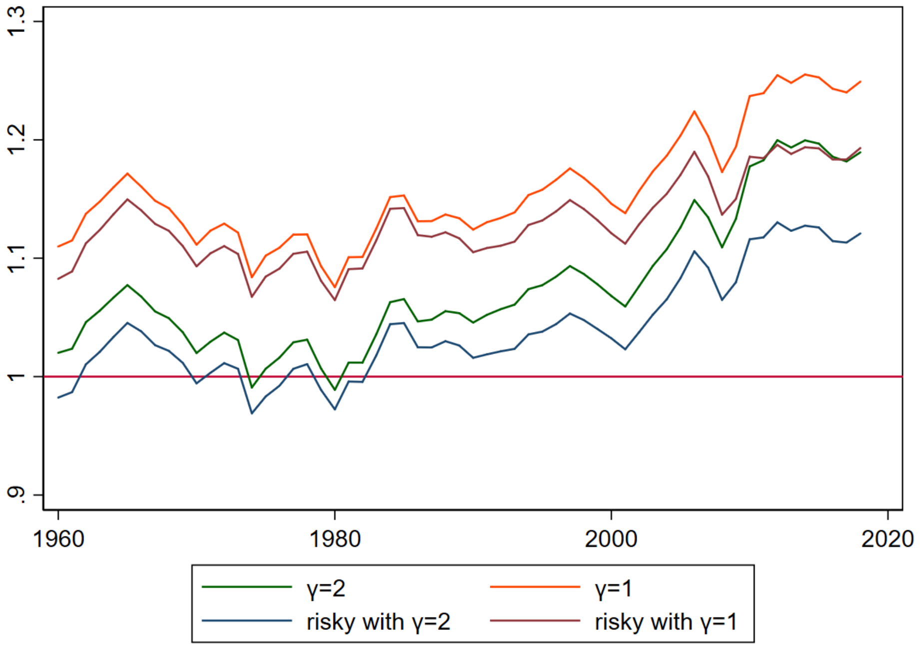
Figure 9: **Smoothed growth rate of TFP**

that our markup is very close to the estimation produced in Fahri and Gourio (2022) whereas they account for the ERP.

6.2 Accounting for labor rents

So far, we followed the literature and assumed that the rents were entirely captured by capital holders. This indeed concurs with recent results analyzing the labor share reaction to different shock, including union decreasing power (Bergholt et al. 2022). In other words, the wages bill share matches the labor share. However, this might not be the case if rents are distributed to workers, as it is often done through incentive bonuses. As such, since part of the wages bill comes from rents, the labor share is always overestimated. To account for this effect, let's η be the share of profits distributed to workers and $1 - \eta$ the share of profits distributed to capital holders. The true labor share is thus given by $S_L = \bar{S}_L - \eta\pi$, with $\bar{S}_L$ being the wage bill share of value added. So, the profit share is now given by:

$$\pi = \frac{(1 - \bar{S}_L - S_K)}{1 + \eta} \tag{22}$$



Note: The figure compares markup estimation based on different measure of the RRK applied to the business sector. The "risky" estimation is based on risky assets return from eq. (21). γ stands for the coefficient of relative risk aversion.

Figure 10: **Markup in the business sector**

Using (9) and (22) we can compute the rent adjusted labor share from:

$$S_L = \bar{S}_L - \frac{\eta}{1+\eta}(1 - \bar{S}_L - S_K) \tag{23}$$

The main difficulty is to know about η which cannot be measured directly and must be estimated. To do so, let's start with the gross operating surplus (GOS) and the wages bill (WB) measure:

$$GOS = (1-\eta)\Pi + rK \tag{24}$$

$$WB = \eta\Pi + wL \tag{25}$$

The rise in capital income (rK) in the very short run is equal to $S_K \mathrm{d}Y$ while the rise in labor income (wL) is equal to $S_L \mathrm{d}Y$.[12] Using the exact differential equation from (24) and (25), then dividing both side by Y, after some algebra, we obtain:

$$\frac{\mathrm{d}WB}{Y} = \frac{\eta}{1-\eta}\left(\frac{\mathrm{d}GOS}{Y}\right) + \left(1 - \frac{S_K}{1-\eta} - \pi\right) g_y \tag{26}$$

Because there are two unknowns here, we cannot directly calculate η. We circumvent the problem by estimating its value and its evolution based on the following regression:

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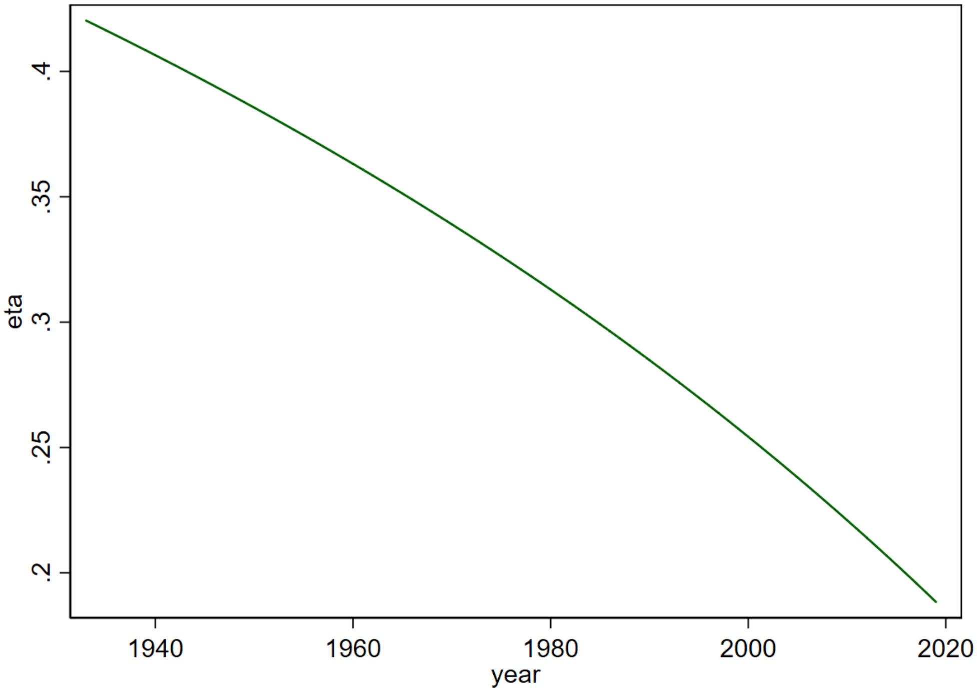
[12] This means to assume that income distribution between labor and capital in the very short run.

$$\frac{\mathrm{d}WB}{Y} = \beta_1 \left(\frac{\mathrm{d}GOS}{Y}\right) + \beta_2 \left(\frac{\mathrm{d}GOS}{Y}\right) \times year + \beta_3 g_y + \varepsilon_t \quad (27)$$

So that $\hat{\eta}_t = \frac{\hat{\beta}_1+\hat{\beta}_2\times year}{1+\hat{\beta}_1+\hat{\beta}_2\times year}$, which produces the coefficient plotted in Figure 11.

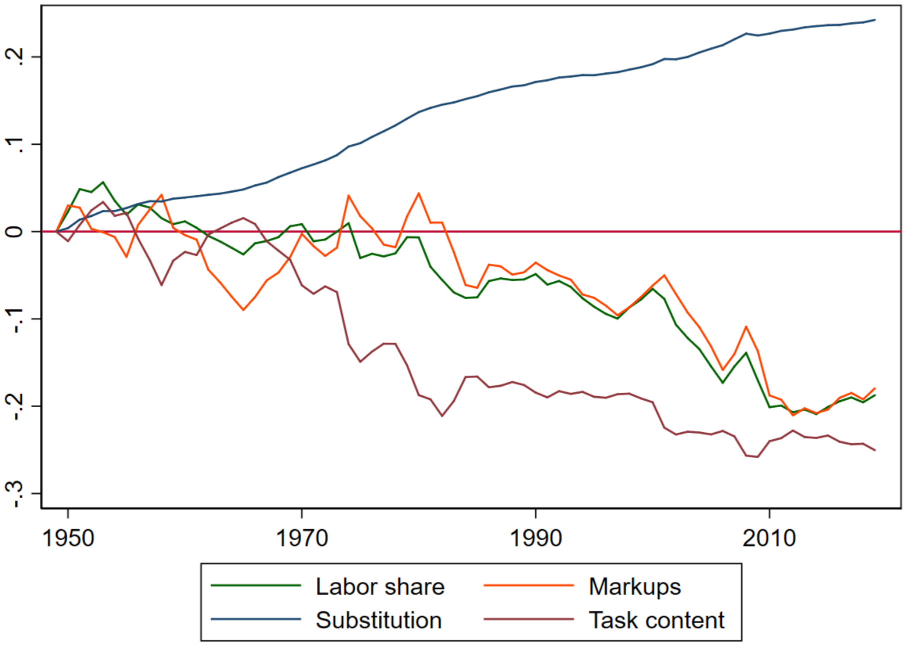


Note: The share of rents distributed to labor (η) is based on OLS estimation from eq. (27) with $\hat{\eta}_t = \frac{\hat{\beta}_1+\hat{\beta}_2\times year}{1+\hat{\beta}_1+\hat{\beta}_2\times year}$.

Figure 11: **Estimation of the share of rents distributed to labor**

According to this estimation, the share of profits distributed to labor declines over the period from 40% in 1933 to 20% in 2019. Based on this result, we can run the same set of calculation as in section 3. Then, we can use equation (22) to come up with the adjusted computation of profits and markups.

The labor share decomposition shows that the decline in the labor share duing the 1970s is mostly due to task content change while its decrease after 2000 is mostly due to markup, although automation still have a decreasing effect (figure 12). This result come directly from the fact that although markups are increasingly large since the 1980s they are less distributed to labor, so the effect of rents is increased by this additional element. For that reason, it is both the rise in markups and the declining power of labor on the sharing of the related rents which explain the labor share decline. This somehow coincides with explanations on the decline of unions and the rising power of firms on labor market. In other words, the decline in the labor share might be explained by the increasing monopolistic and monopsonistic power of enterprises.



Note: The labor share change is decomposed based on eq. (15) to account for the change in task content, markup and capital-labor substitution effect. Here the labor share calculation remove the rents distributed to labor based on the estimation of eq. (27). g_A (the growth rate of A_L/A_K) is assumed to be equal to the growth rate of labor productivity. The relative risk aversion is fixed to 0.71 while the elasticity of substitution between labor and capital is fixed to 0.8.

Figure 12: **Labor share decomposition with the new markup measurement**

7 Discussion

The set of measures we produced so far depends on the hypotheses used to estimate the RRK. In particular, one can question the form of the utility function as we have to assume that the rate of relative risk aversion is equal to the inverted inter-temporal elasticity of substitution and fixed this parameter along with the discount factor. The alternative would be to make use of a macroeconomic model as in Fahri and Gourio (2018) but this comes at the cost of other hypotheses. In fact, Fahri and Gourio (2018) results is "essentially the calibration of the steady-state of a very bare-bone DSGE model" which dwells on a Cobb-Douglas production function. Besides, the choice of targeted moments plays a decisive role on the result of this exercise.[13] For that reason our result should be viewed as complementary to this approach.

The first way to address those issues is to extend our calculation based on more complex utility function. The second way to address this critic is to look at the global coherency of our estimations according to its capacity to explain some of the "big ratios" used in the literature. As such we proceed in the opposite direction compared to Fahri & Gourio (2018) as we do not use those ratios as target moments but as a validity test.

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[13] For instance, the use of TFP growth implies some assumption about parameters value (*e.g.* α) which may enter in contradiction with the estimation of these parameters in this very exercise.

7.1 Expectations

The result so far did not account for expectation, although we know that consumption volatility and the covariance between consumption growth and the RRK can affect the result. If the utility function is:

$$U = \sum_{t=0}^{+\infty} \beta^t E_t[u(c(t))] \tag{28}$$

then the related Euler equation becomes:

$$u(c_t) = \frac{1}{1+\rho} E_t[Ru(c_{t+1})]$$

Assuming that the expected taxation rate is independent to the RRK and taking a second-order approximation to marginal utility around $R = g_c = 0$, gives:

$$E_t[R] = \frac{\rho + E_t[\varphi_h] + \gamma E_t[g_c] - \frac{1}{2}\gamma(\gamma+1)(E_t[g_c]^2 + V[g_c]) + \gamma Cov[R, g_c]}{(1 - \gamma E_t[g_c])(1 - E_t[\upsilon_a])} \tag{29}$$

To measure the covariance we will use the rolling covariance between consumption growth and the rate of return on everything calculated by Jordà et al. (2017). As a matter of fact, the related covariance is extremely weak. Over the whole period the covariance coefficient is equal to 0.00055.[14] This means that $\frac{1}{2}\gamma(\gamma+1)(E_t[g_c]^2 + V[g_c]) > \gamma Cov[R, g_c]$, suggesting that our previous calculation overestimate the RRK. However, the gap is small, as shown by figure 13, which compare the RRK based on the mere and the expected utility function.

7.2 Intertemporal elasticity and relative risk aversion

Another issue is the use of a non-recursive utility function. This prevents from distinguishing between the elasticity of intertemporal substitution and the relative risk aversion, so that $1/\psi = \gamma$. The use of recursive preferences is not without costs though, as it rarely lead to simple solutions. However, in some rare cases the Epstein and Zin (1989) utility function allows to calculate the RRK from the consumption growth. This gives us the opportunity to assess the use of non-recursive utility function in our calculation.
The Epstein and Zin utility function is define as:

$$U_t = \left[(1-\beta)c_t^{1-1/\psi} + \beta\left(\mathbb{E}_t U_{t+1}^{1-\gamma}\right)^{\frac{1-1/\psi}{1-\gamma}}\right]^{\frac{\psi}{\psi-1}}$$

The related Euler equation is:

$$\mathbb{E}_t\left[\left(\beta\left(\frac{c_{t+1}}{c_t}\right)^{-1/\psi}\right)^{\frac{1-\gamma}{1-1/\psi}} (R_{t+1})^{\frac{1-\gamma}{1-1/\psi}}\right] = 1$$

We immediately see that $1/\psi = \gamma \Rightarrow \frac{1-\gamma}{1-1/\psi} = 1$ which is the usual CRRA function. This Euelr equation prevents from isolating R_{t+1} as far as both consumption growth and the RRK have a

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[14] This coincides with previous estimations since Mehra and Prescott (1985).

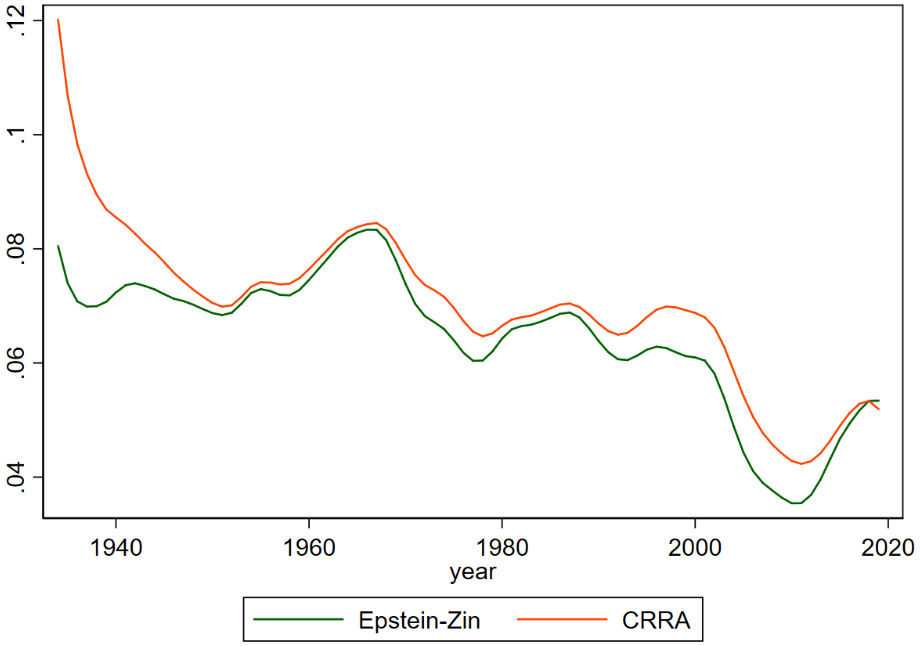
Note: The RRK for the expected consumption risk is based on $\gamma = 1/\psi = 2$.

Figure 13: **RRK with expected consumption risk**

stochastic component. In fact, if we define $R_t = \bar{R} + \epsilon_t$ with ϵ_t a white noise with $\mathbb{E}[\epsilon] = 0$, and, $\frac{1}{1+g_c} = \mu + \theta_t$ with θ_t a white noise with $\mathbb{E}[\theta] = 0$, $\bar{R}$ cannot be isolated for all values of ψ and γ. However, there is one case providing exact solution as $\psi = 2/3$ and $\gamma = 2$ (see the appendix for the related algebra). This case is particularly interesting since the value of the EIS and RRA are in line with analyzes assessing their value (see Elminejad et al. (2023)). Comparison with the CRRA can then help us to assess the potential bias from our previous calculation.
Figure 14 compares the Epstein and Zin recursive utility function with the one produced from a simple CRRA function with $\psi = 2/3$. In both cases we ignore taxes and intermediation costs as our principal objective is to measure the consequence of the use of non-recursive utility function. The results shows that the Epstein and Zin case produce slightly lower RRK but with a similar trend. This means that the CRRA simplification has no effect on the results produced so far.

7.3 Explaining big ratios

The first implied ratio our calculation allows to deal with is the capital risk premium (KRP). Based on equation (21) we set $KRP = m - r$ which corresponds to $KRP = \left(1 + \frac{d}{a}\right)(R - r)$. We can compare our results with empirical measures of the ERP based on different methods (Gordon, Fama-French, Campbell and Thompson) as done in Fahri and Gourio (2018). As a matter of fact, our measure of the capital risk premium follows very similar pattern with a strong increasing value from 1990 to 2010 (figure 15). In addition, the series obtained based on $\gamma = 1$ is comprised from 2% to 8% and coincides with empirical estimation of the ERP. In this



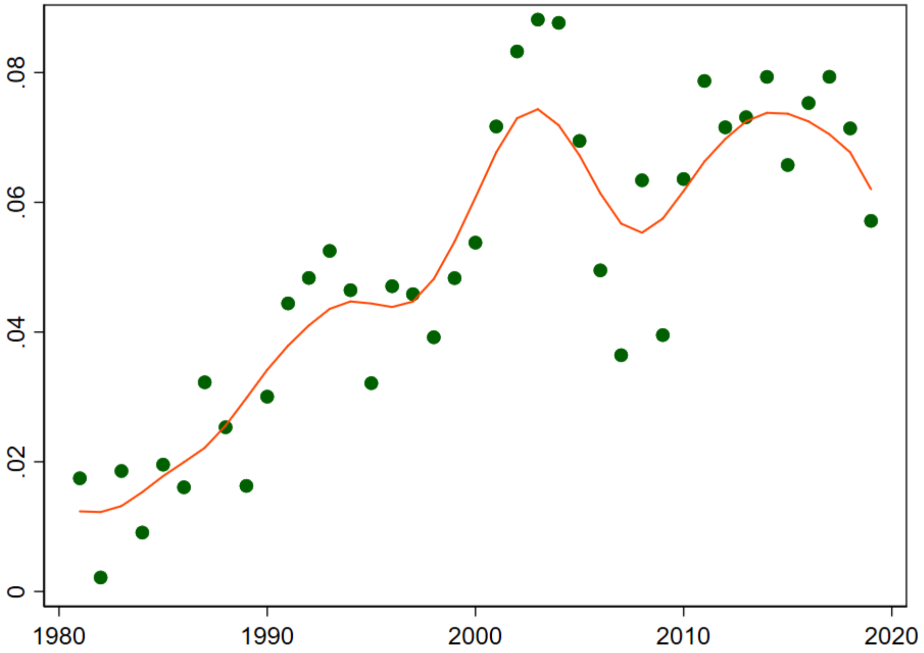
Note: The RRK calculation from Epstein and Zin utility function is based on $\gamma = 2$ and $\psi = 2/3$. Calculation for the CRRA is based on $\psi = 2/3$ (or $\gamma = 3/2$).

Figure 14: **RRK with recursive utility function**

respect, our induced measure of the KRP is fully in line with empirical data.
The other figures discussed in the literature on the topic are the investment/capital ratio and the Tobin's Q. Investment has been low over the past 15 years despite the drop in interest rates. This fact coincides with our results for different reasons. First, the rise in markups has caused an increasing wedge between the marginal product of capital and the cost of capital, thereby reducing the incentive to invest. In addition, as showed by our measure of the KRP, the cost of capital did not decrease as much as the interest rate. For that reason, the concomitant rise in markup more than offset the drop in lending rates. This also coincides with the decrease in Tobin's Q and the rise in wealth to output ratio, as documented in Eggertson et al. (2021).

8 Conclusion

We have shown in this study how important is the measure of the RRK. Knowledge about its value opens a large space for the measurement of fundamental macro variables among which the national markup, the change in task content, the substitution effect or total factor productivity. In so doing, our measurement exercise helped produce new facts which allows to distinguish between the numerous explanations and hypotheses on the change in value added distribution over the past 20 years. Our main result is that the markup and the change in task content in favor of capital (mostly due to automation) rise over the past 20 years, thereby decreasing the labor share. Meanwhile, the decline in the TFP growth during the same period suggests that these phenomena are not necessarily related to an improvement in welfare, at least in the



Note: The capital risk premium is based on the difference between the risky RRK (m) from equation (28) and the risk free rate (r). The calculation is based on $\gamma = 0.97$.

Figure 15: **Induced capital risk premium**

middle run.

However, some issues still need to be addressed. In particular, the analysis of the markup has to better account for the distribution of the related rents between workers and capital holders. Although our study proposes to approximate its evolution over time based on econometric estimations, a direct measure still needs to be done. By the same token, since our inquiry provides long term data on the trend of the RRK, one might question some of the hypotheses used in the computation exercise, such as fixed discount factor and intertemporal elasticity. Although the set of results produced here remains coherent with these choices, additional information on the evolution of those parameters might have consequences for comparisons in a very long time horizon.

Lastly, this study opens new perspectives on topics for which the RRK, the value added distribution, and the productivity are at stake. This includes, among others, the analysis of inequality based on the $r - g$ hypothesis (Piketty, 2013; Jorà et al., 2019); the sustainability of the public debt since the RRK directly affects debt revenue (Reis, 2022a); the automation and offshoring of tasks (Acemoglu and restrepo, 2018, 2020; Dao et al., 2022); inflation and monetary policy—given the effect of real interest rates change on prices (Cochrane, 2022); economic growth—because markup and innovation might be tied up in the long run (Aghion et al., 2019). More theoretically, markup and RRK data can also be used in macroeconomic model calibration and estimation, such as done in Reis (2022a) on debt revenue. As such, our inquiry might be seen as a contribution toward additional analysis on these challenging topics.

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